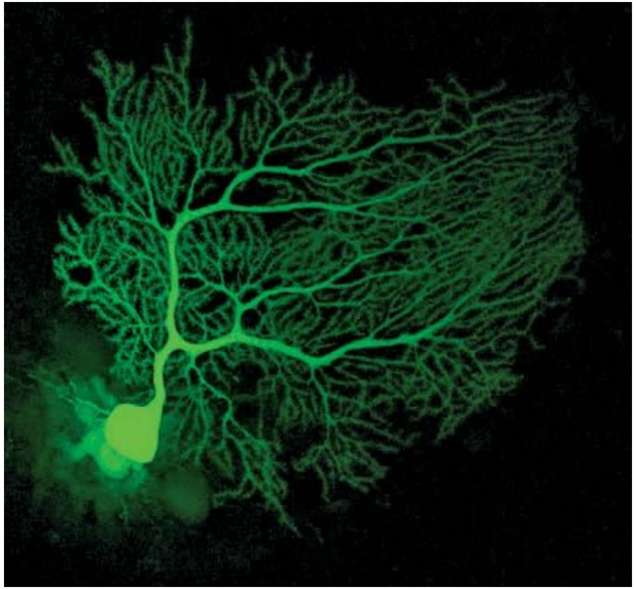
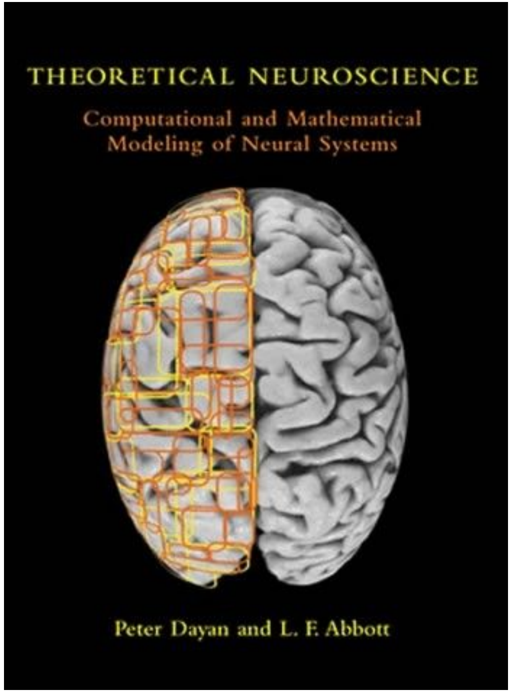
Neural encoding



Aukkawut Ammartayakun and Gabrielle Plainte

Reference

- Chapter 1 Neural Encoding I: Firing Rates and Spike Statistics



Spike-Triggered Average

We can approximate the average stimulus over an interval as

Average of sum of the response

$$C(\tau) = \left\langle \frac{\sum_{i=1}^{n} s\left(t_i - \tau\right)}{n} \right\rangle \approx \frac{\left\langle \sum_{i=1}^{n} s\left(t_i - \tau\right) \right\rangle}{\langle n \rangle}$$

Average of number of spikes per trials

Or we can find the average from the integral method.

$$C(\tau) = \frac{1}{\langle n \rangle} \int_0^T dt \, \langle \rho(t) \rangle \, s(t - \tau)$$

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Window size of trials

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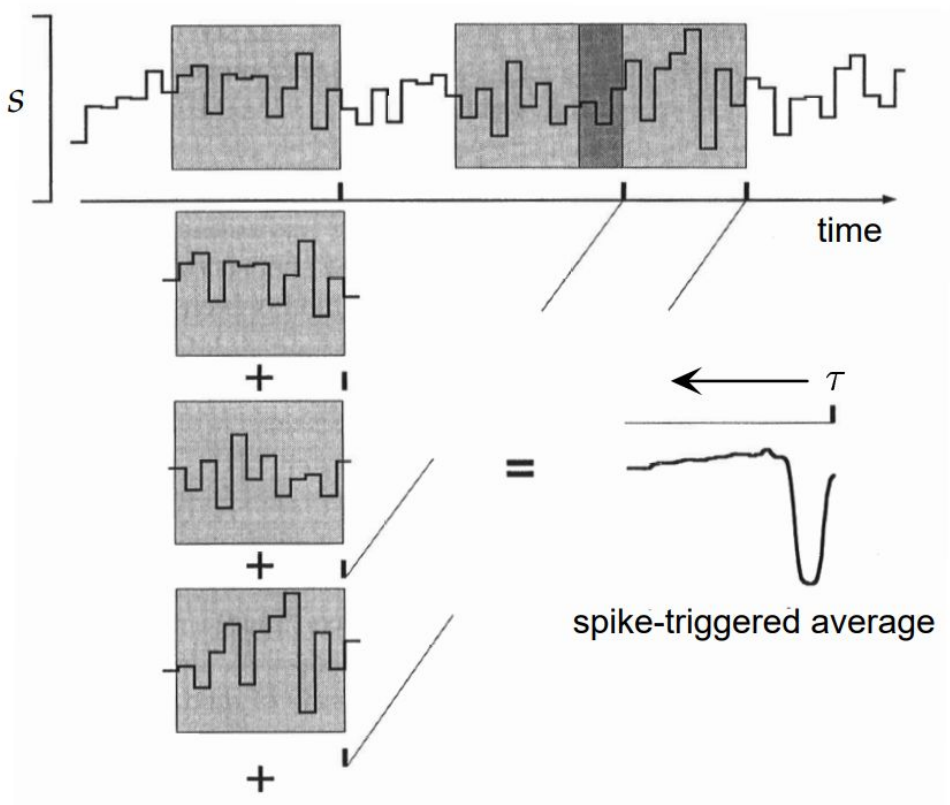
Equivalent to $r(t)$

Spike-Triggered Average

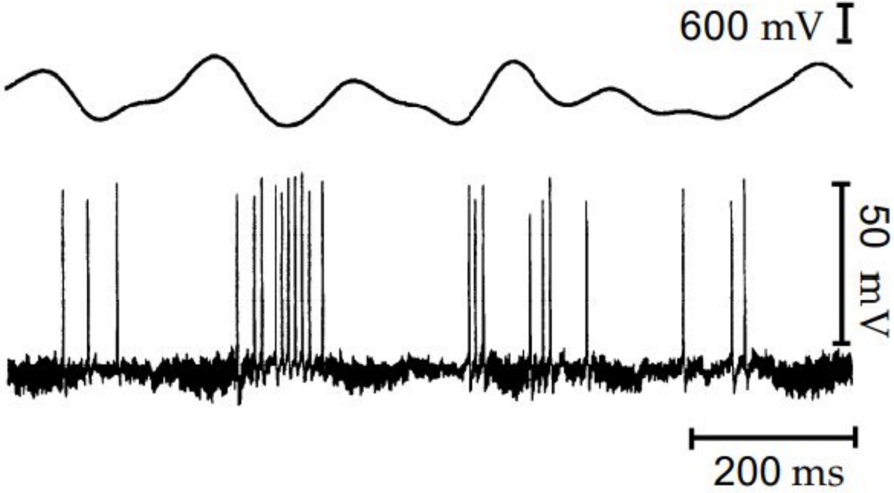


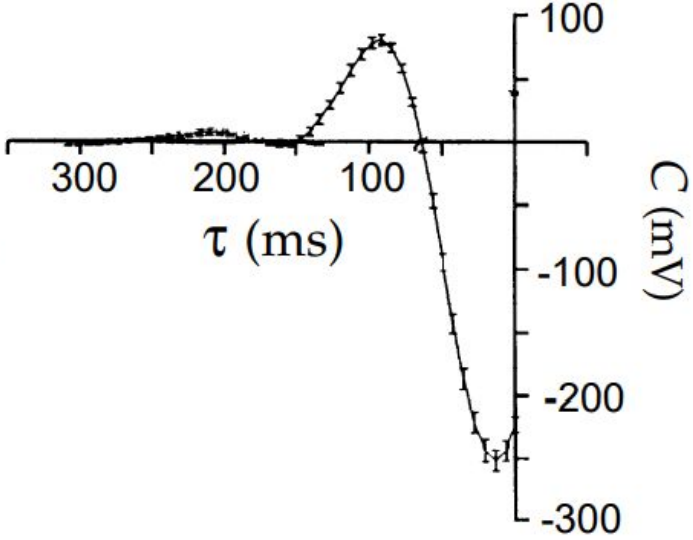
Correlation of Firing Rate Stimulus

As we know the spike-triggered average, we can define the correlation of firing rate stimulus as

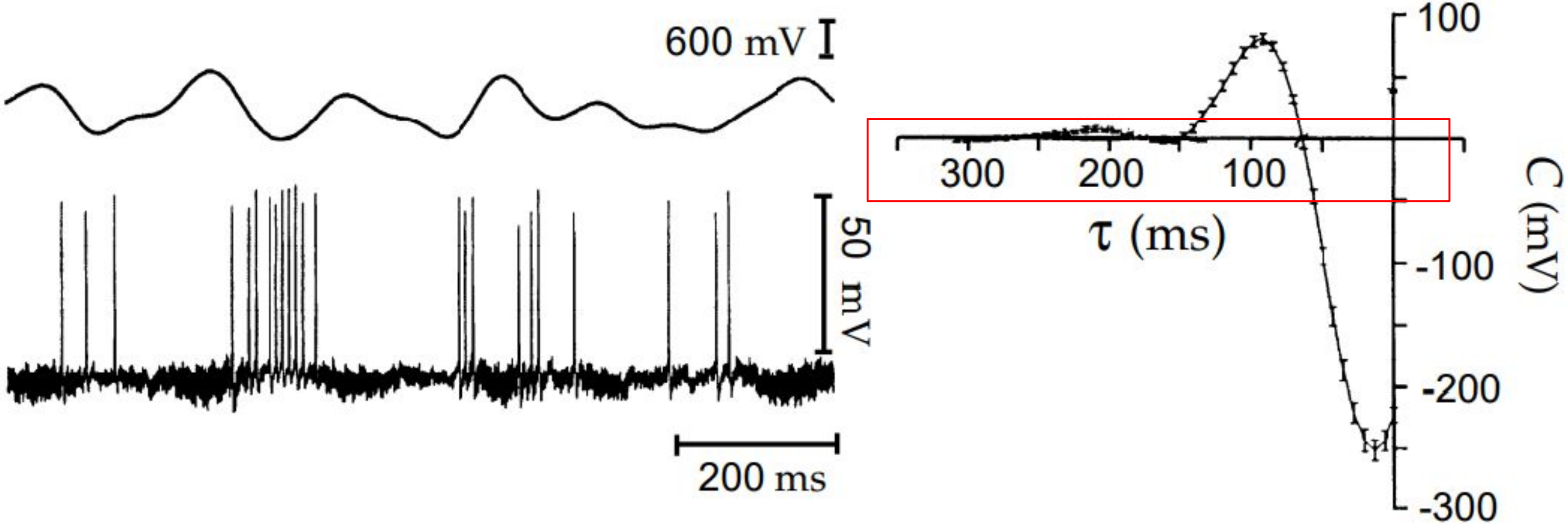
$$Q_{rs}(-\tau) = \frac{\langle n \rangle}{T} C(\tau)$$

Correlation of Firing Rate Stimulus



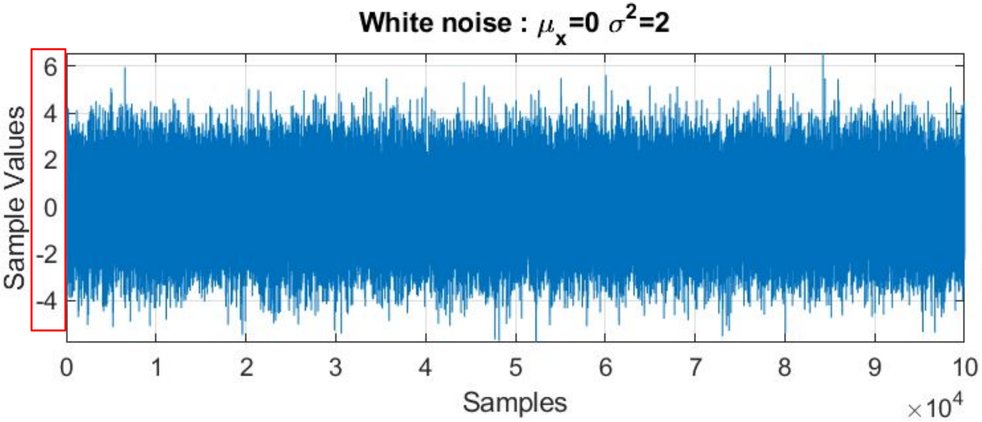


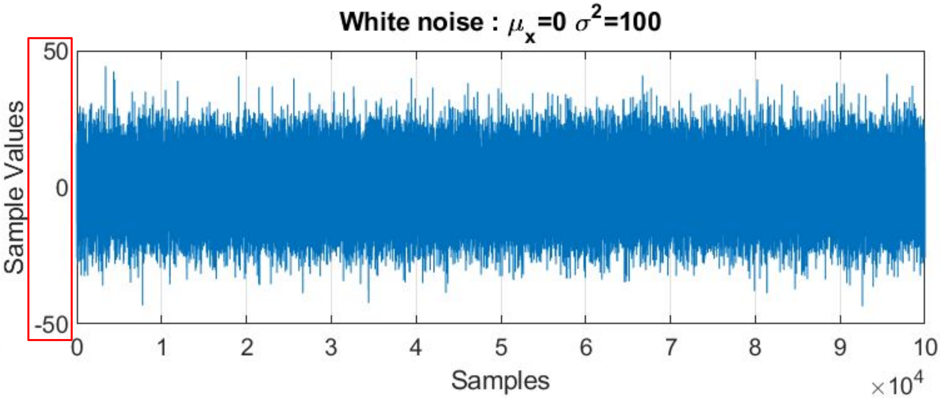
Correlation of Firing Rate Stimulus



White-Noise Stimuli

Spike-triggered average is depending on the stimulus. We want to use the signal that is not correlated from time to time.

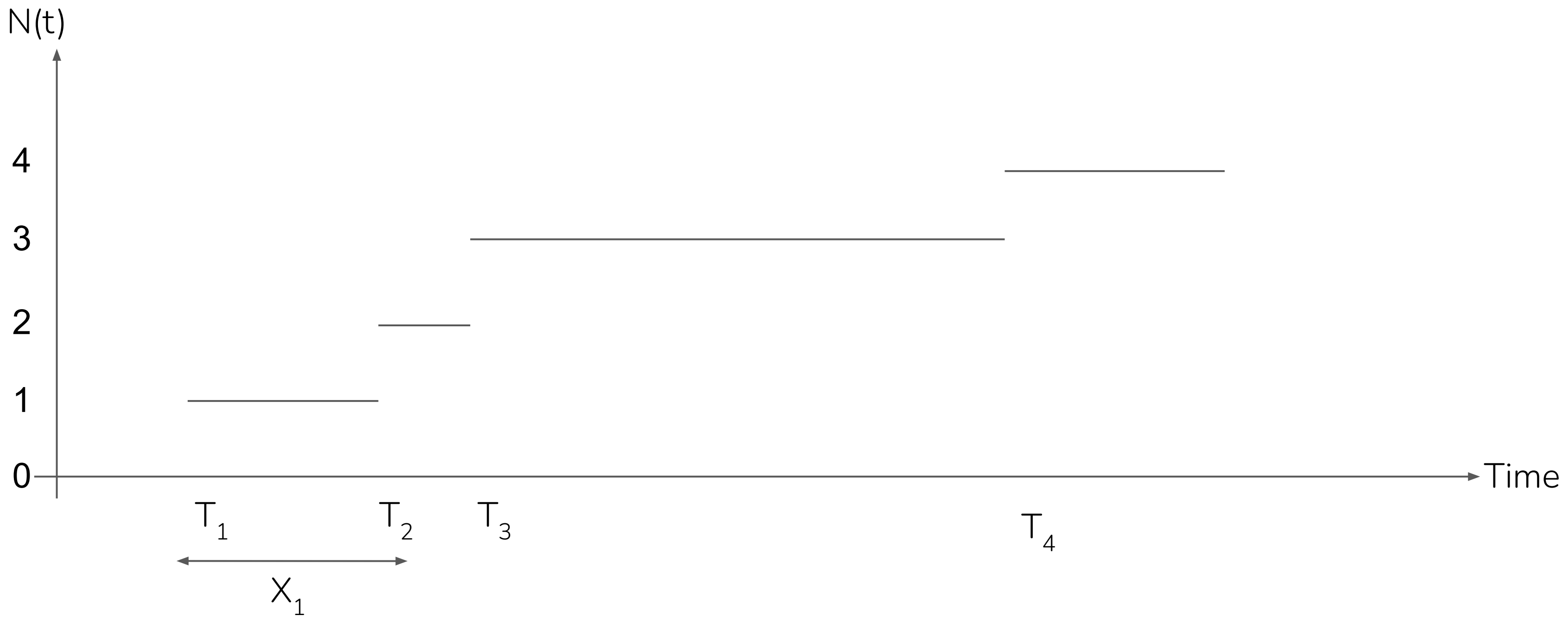




$$Q_{ss} = \sigma_s^2 \delta(\tau) = \frac{1}{T} \int_0^T dt s(t) s(t+\tau)$$

Stochastic Process

- Point process (or counting process)



Stochastic Process

Question:

How can we define the counting process $N(t)$? What assumption do we need?

Stochastic Process

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How can we define the counting process N(t)? What assumption do we need?

Independence of sequence of r.v. X and sequence of r.v. T

Poisson process

Let say the fire rate of neuron is $r(t) = r$ (homogeneous)

Probability of that exactly n spikes occurs within the duration **τ**

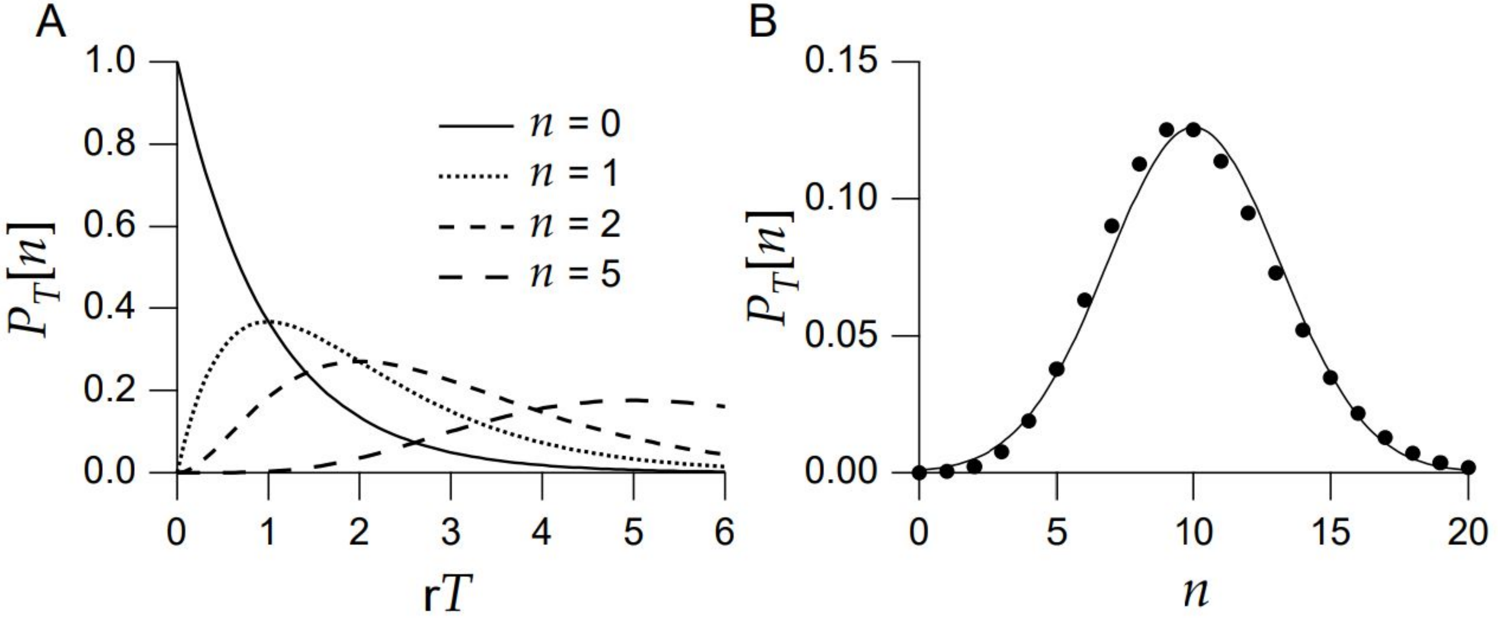
$$\mathbb{P}\left(T_1, T_2, \ldots\right) = n! P_\tau(n) \left(\frac{\Delta t}{\tau}\right)$$

Probability of spike sequence

Note that $P_\tau(n) \sim \mathrm{Poi}(r\tau)$.

(Proof can be found on textbook pg. 25 - 26)

Poisson process



$$P_\tau(n) \sim \mathrm{Poi}(r\tau)$$

Interspike Interval Distribution

Let say we want to calculate the probability density of time intervals between spikes (interspike interval) for determining spiking pattern. This can be done with multiplication rule of probability

$$\mathbb{P}\left(\tau \leq t_{i+1} - t_i \leq \tau + \Delta t\right) = r\Delta t e^{-rT}$$

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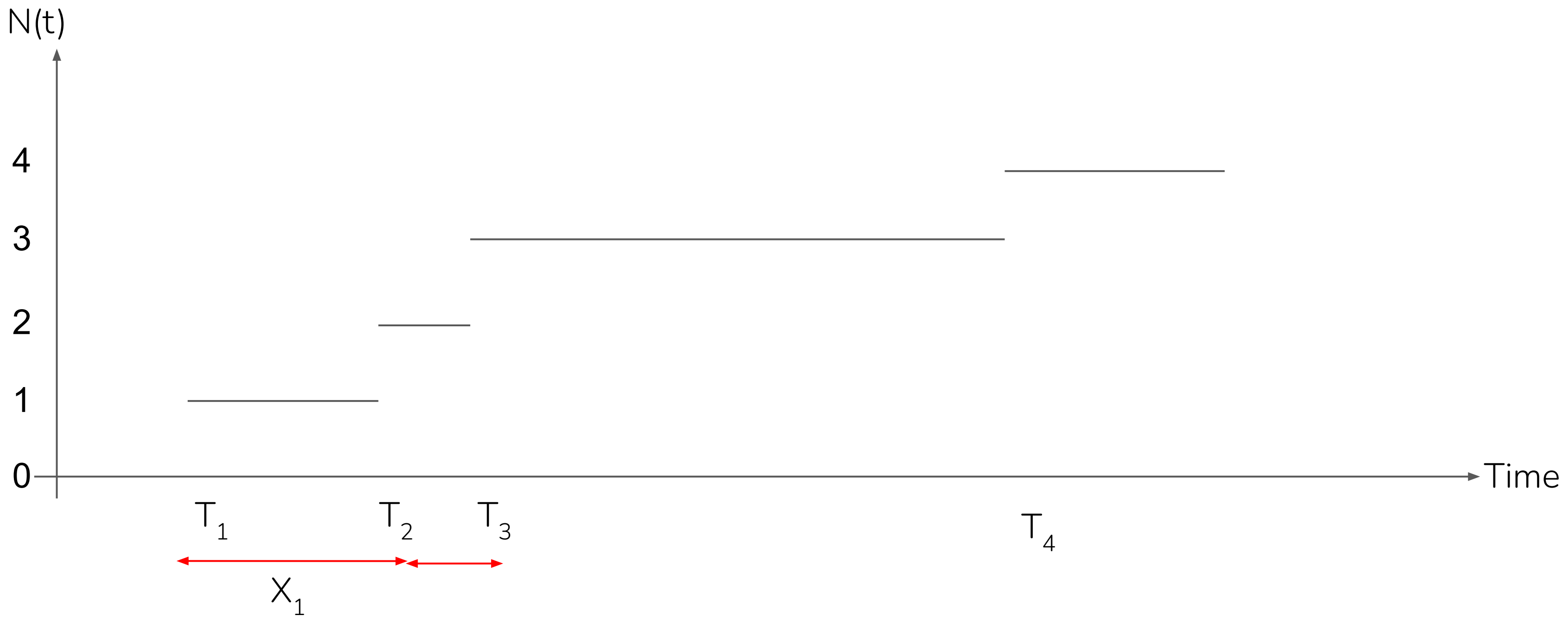
With expected value $\langle \tau \rangle = \int_0^\infty d\tau \tau r e^{-rt} = \frac{1}{r}$

and variance $\sigma_\tau^2 = \left[\int_0^\infty d\tau \tau^2 r e^{-rt}\right] - \langle \tau \rangle^2 = \frac{1}{r^2}$

Stochastic Process

- Point process (or counting process)



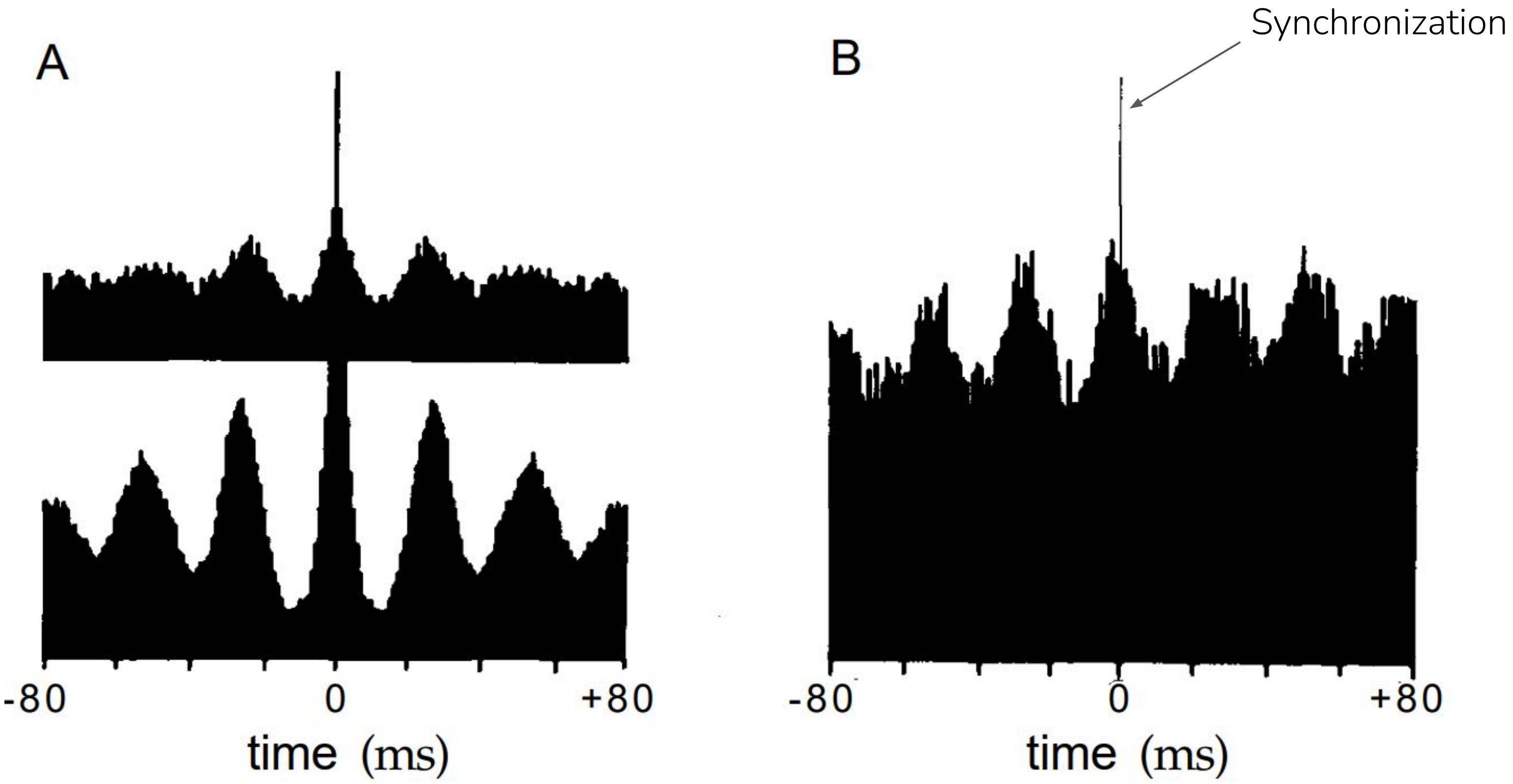
Spike-Train Autocorrelation

$$Q_{\rho\rho}(\tau) = \frac{1}{T}\int_0^T dt\,\langle(\rho(t) - \langle r\rangle)\,(\rho(t+\tau) - \langle r\rangle)\rangle = \frac{1}{T}\int_0^T dt\,\left\langle\left(\rho(t) - \frac{\langle n\rangle}{T}\right)\left(\rho(t+\tau) - \frac{\langle n\rangle}{T}\right)\right\rangle$$

With homogeneous poisson process assumption ($\langle r\rangle = r$),

$$Q_{\rho\rho}(\tau) = r\delta(\tau)$$

Cross-Correlation



Inhomogeneous Poisson Process

What if firing rate is not constant but the function of time?

$$\mathbb{P}\left(t_1, t_2, \ldots, t_n\right) = \exp\left(-\int_0^T dt r(t)\right) \prod_{i=1}^{n} r\left(t_i\right)$$

Poisson Spike Generator

$$D(x_{rand}, t) = \begin{cases} \text{fire} & r_{est}(t)\Delta t > x_{rand} \\ \text{not fire} & \text{otherwise} \end{cases}$$

Question: What about refractory period?

Poisson Spike Generator

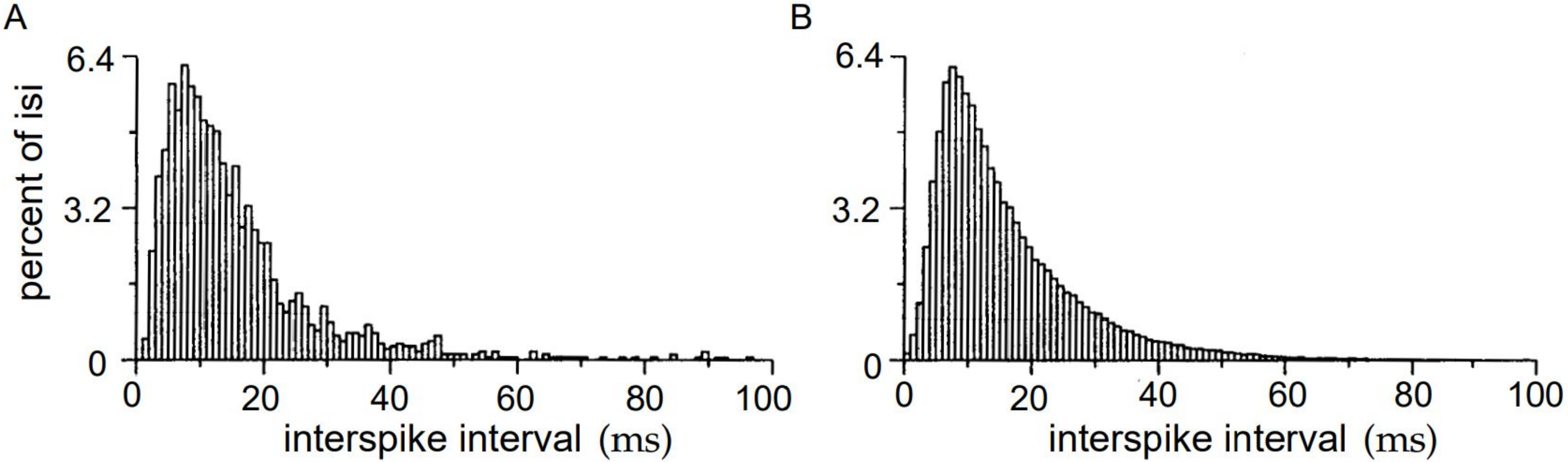
$$D(x_{rand}, t) = \begin{cases} \text{fire} & r_{est}(t)\Delta t > x_{rand} \\ \text{not fire} & \text{otherwise} \end{cases}$$

Question: What about refractory period?

Set r(t) = 0 after firing the neuron. Then, recover it exponentially.

Fit with different model.

Goodness of fitting



Neural Code

Rate coding vs Temporal coding

- High fluctuation on frequencies of firing.
    - Either noise or encoding information.

Neural Code

Rate coding vs Temporal coding

- High fluctuation on frequencies of firing.
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Question

Does individual action potentials independent?

Independent-Spike Coding

Let's look into the inhomogeneous poisson process,

$$\mathbb{P}\left(t_1, t_2, \ldots, t_n\right) = \exp\left(-\int_0^T dt r(t)\right) \prod_{i=1}^{n} r\left(t_i\right)$$

Independent-Spike and Correlation Coding

Let's look into the inhomogeneous poisson process,

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Time dependency

If the information generated by neuron does encode independently, we called it ***independent-spike coding***. However, if that is not the case, we called it ***correlation coding***.

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Time dependency

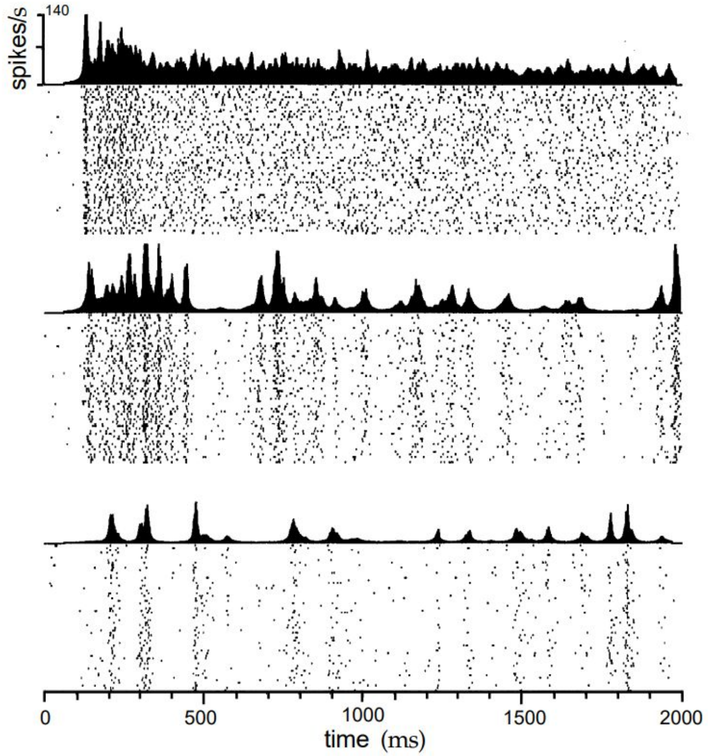
If the information generated by neuron does encode independently, we called it ***independent-spike coding***. However, if that is not the case, we called it ***correlation coding***. What about the whole neuron? What about synchronization between neurons?

Temporal Code

Let say we want to extract the information out of neuronal response, **how precise a measurement should be?**

- Does those high frequencies fluctuation carry information?
    - Does that occurs because of the nature of the stimulus?

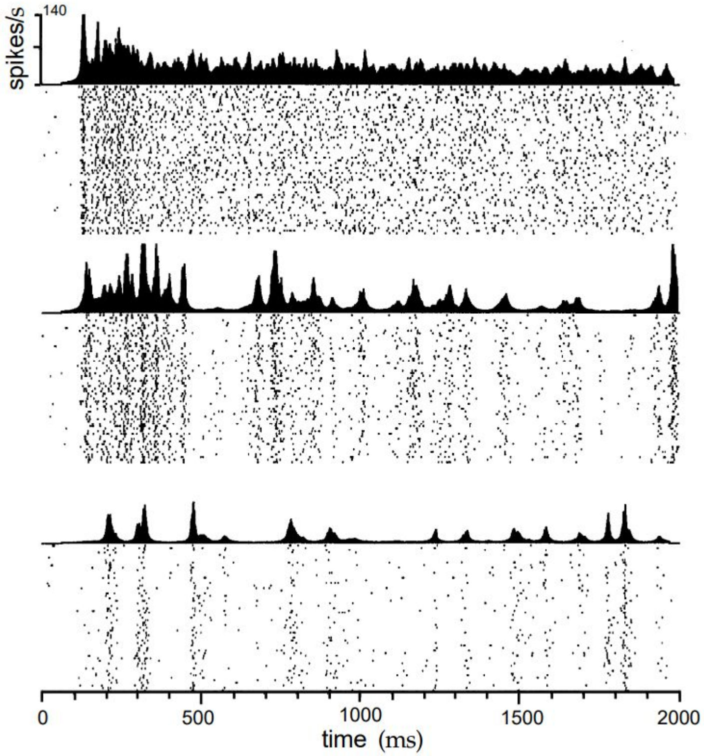
Temporal Code



Rate coding

Temporal coding

Temporal Code

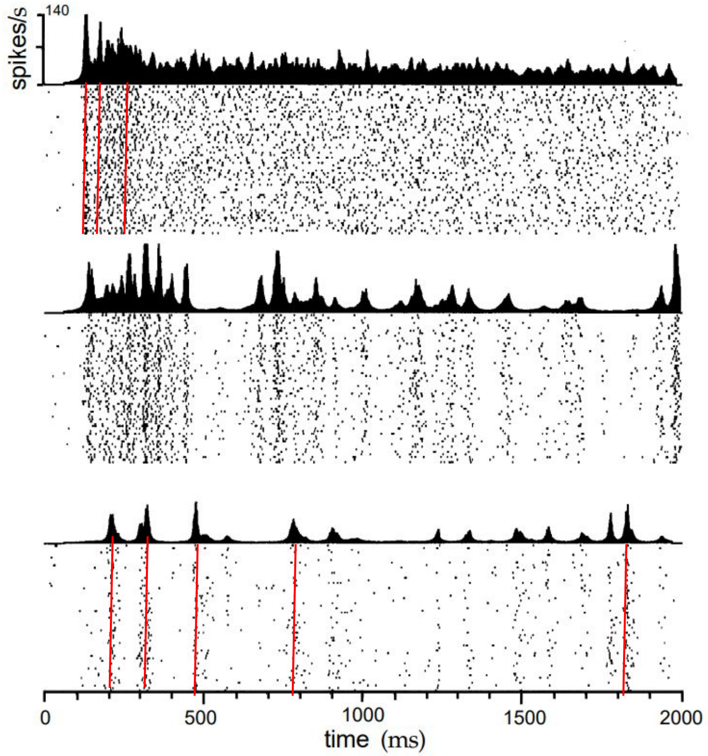


Rate coding

What r(t) explicitly express rate/temporal behavior?

Temporal coding

Temporal Code



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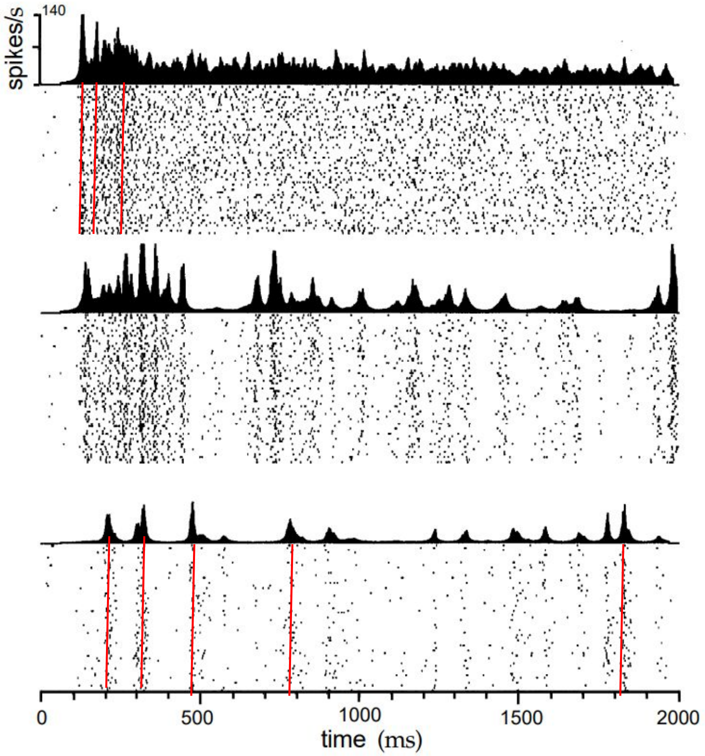

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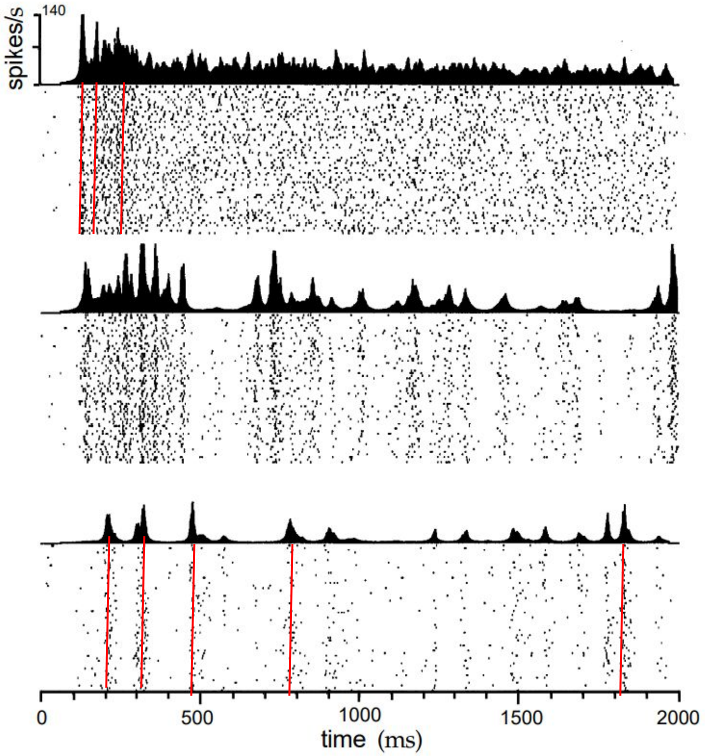

Rate coding

What r(t) explicitly express rate/temporal behavior?
What about collective decision?
Fourier decomposition of r(t)?

Temporal coding